Lecture notes on Dynamic Meteorology prepared by Somenath Dutta for old advanced course.

Chapter-V

Atmospheric energetics

By the term atmospheric energetics, we understand the different forms of energy, the atmosphere possesses and conversion between them.

Atmosphere possesses energy mainly in three forms, Viz., the internal energy, the kinetic energy and potential energy.

Atmospheric internal energy: It is due to heating of the atmosphere. To obtain an expression for global atmospheric internal energy, let us consider unit mass at temp T^0 K. Then internal energy of this unit mass is $C_V T$. Now consider an infinitesimal volume '$d\sigma$' with density 'ρ' of the atmosphere. This volume is so small that the density ρ practically remains invariant in it. So its mass is $\rho\, d\sigma$. So, the internal energy of this infinitesimal volume is $C_V T \rho\, d\sigma$.

Hence the internal energy of the global atmosphere is $\iiint_\sigma \rho C_V T d\sigma$ = I , say.

Atmospheric potential energy: It is due the vertical position of the centre of gravity of the atmosphere. The potential energy of unit mass at a height 'z' above the mean sea level is gz. Hence following the same argument as in I.E, we have the expression for potential energy of global atmosphere as $\iiint_\sigma g \rho z d\sigma$ = P, say.

Atmospheric kinetic energy: The kinetic energy of the atmosphere is due to different atmospheric motion. Kinetic energy of an unit mass moving with velocity '$\vec{v}$' is $\frac{\vec{v}.\vec{v}}{2}$.

Hence the expression for kinetic energy for global atmosphere is $\iiint_\sigma \rho \frac{\vec{v}.\vec{v}}{2} d\sigma$ =K, say.

Energy equations:

The global ***internal energy*** equation is given by

$\frac{dI}{dt} = \iiint_\sigma \rho \dot{Q} d\sigma - \iiint_\sigma p\left(\vec{\nabla}.\vec{v}\right) d\sigma$ ….(1), where, $\dot{Q} = \frac{dQ}{dt}$ represents the rate of heating.

The above equation tells us that the change in global internal energy is due to net heating/cooling of the atmosphere and due to divergent/convergent motion in the atmosphere. Net heating/ cooling leads to an increase/decrease in temperature, which again leads to an increase/decrease in global I.E.

Now to understand how divergence/convergence leads to an increase/decrease in I.E, let us see the following flow charts:

Divergence→Expansion→Cooling→Fall in I.E. On the other hand,

Convergence→ Compression→Heating→Rise in I.E.

The global ***potential energy*** equation is given by,

$\frac{dP}{dt} = \iiint_\sigma g \rho w d\sigma$ ….(2). This equation tells that any change in global potential energy is due net vertical motion. Net rising/sinking motion leads to an increase/decrease in P.

The global ***kinetic energy*** equation is given by,

$$\frac{dK}{dt} = -\iiint_\sigma \vec{\nabla}.(p\vec{v})d\sigma + \iiint_\sigma p\left(\vec{\nabla}.\vec{v}\right)d\sigma - \iiint_\sigma g\rho w d\sigma + \iiint_\sigma \rho\vec{F}.\vec{v}\,d\sigma \;\ldots.(3)$$

In the above equation the first term on the right hand side represents the convergence of flux of energy due to work done by/against the pressure force and the last term represents the rate of work by frictional or dissipative forces. Other two terms, viz., the second and third terms have already appeared in the global internal energy equation and in global potential energy equation respectively, but with respective opposite sign. The last term represents the destruction of K.E due to work done against the dissipative forces.

Again from Gauss divergence theorem, we know that $\iiint_\sigma \vec{\nabla}.(p\vec{v})d\sigma = \iint_S p v_n\, ds$, where, v_n is the component of $\vec{v}$ normal to the surface 'S' enclosing the volume 'σ'. If the global atmosphere is considered to be an isolated closed system, then, $v_n = 0$. So the first term in equation becomes zero.

From equation (1) and (3) it follows that for a given sign of $\iiint_\sigma p\left(\vec{\nabla}.\vec{v}\right)d\sigma$ global kinetic energy / global internal energy will be generated at the expense of global internal energy / global kinetic energy. So, this term may be thought of representing the conversion of internal energy to kinetic energy. We denote it by C(I,K).

Hence, $C(I,K) = \iiint_\sigma p\left(\vec{\nabla}.\vec{v}\right)d\sigma$ or $C(K,I) = -\iiint_\sigma p\left(\vec{\nabla}.\vec{v}\right)d\sigma$.

Already we have seen how divergence or convergence results in decrease or increase in I.E. Also we know that due to divergence or convergence, downstream wind speed increases or decreases, i.e., global K.E increases or decreases. Thus conversion between these two forms of atmospheric energy is due to the divergent flow of the atmosphere.

Similarly from equations (2) and (3), it follows that for a given sign of $\iiint_\sigma g\rho w d\sigma$ global kinetic energy / global potential energy will be generated at the expense of global potential energy / global kinetic energy. So, this term may be thought of representing the conversion of kinetic energy to potential energy. We denote it by C(K,P).

Hence, $C(K,P) = \iiint_\sigma g\rho w d\sigma$ or $C(K,I) = -\iiint_\sigma g\rho w d\sigma$. We have already seen that a net upward or downward motion leads to an increase or decrease in global P.E. Also a net upward or downward motion causes convergence or divergence, which again leads to decrease or increase in global K.E. Thus conversion between these two forms of atmospheric energy is due to net vertical motion in the atmosphere.

Adding equations (1), (2) and (3) we obtain,

$$\frac{dE}{dt} = \iiint_\sigma \rho\dot{Q}\,d\sigma + \iiint_\sigma \rho\vec{F}.\vec{v}\,d\sigma \;\ldots.(4), \text{ where, } E = I + K + P.$$

In equation (4), the first term represents the generation of internal energy by net heating of the atmosphere and the second term represents the destruction of K.E due to work done against the dissipative forces. They are respectively denoted by $G(I)$ and $-D(K)$. Thus equation (4) may be written as

$$\frac{dE}{dt} = G(I) - D(K) \ldots .(5).$$

From equation (5) it is clear that for the source of total atmospheric energy, $G(I)$ should be positive and more, i.e., there must be net heating in the atmosphere. Again net heating is by Solar energy. ***Thus the Sun is source of all atmospheric energy.***

Now let us consider the kinetic energy equation for horizontal motion. First of all $w = 0$. So, we have, $\frac{dK_h}{dt} = S(K) - D(K)$. Here, $K_h = \frac{\vec{v}_h.\vec{v}_h}{2}$ is the kinetic energy for horizontal motion and $S(K) = \iiint_\sigma p\left(\vec{\nabla}.\vec{v}_h\right)d\sigma$. Now question which region should be a source for horizontal kinetic energy. Now in a source region for horizontal kinetic energy there should be production of horizontal kinetic energy, i.e., $\frac{dK_h}{dt} \rangle 0$. Again for $\frac{dK_h}{dt}$ to be positive, $S(K) = \iiint_\sigma p\left(\vec{\nabla}.\vec{v}_h\right)d\sigma$ should be positive and large, which requires p to be high and $\vec{\nabla}.\vec{v}_h \rangle 0$. This conditions exist in the region of sub-tropical anticyclone which is characterized by high pressure and divergence. ***Thus the belt of sub-tropical anticyclone is the source for horizontal kinetic energy.***

Energetics in a hydrostatic and stably stratified atmosphere:

By the *hydrostatically stable atmosphere* we simply understand that there is no net vertical acceleration and by *stable stratification* we understand that in the atmosphere heavy colder air is below the light warmer air or the potential temperature (θ) increases with height. Now it will be shown that in such an atmosphere internal energy is proportional to potential energy. This will be established by showing below that any change in I.E causes a similar change in P.E and vice-versa.

Increase or decrease in I.E→Increase or decrease in Temperature (T)→Expansion or contraction of an air column of unit cross sectional area→Rising or sinking motion→Increase or decrease in P.E. Similarly,

Increase or decrease in P.E→ Rising or Sinking motion→Convergence or divergence→Increase or decrease in I.E.

Thus any change in I.E causes a similar change in P.E and vice-versa. Hence internal energy is proportional to potential energy.

The above can be established mathematically also as shown below:

We consider an air column with unit cross-sectional area. The P.E of this air column is given by

$$P = \int_0^\infty g\,\rho\,z\,dz = -\int_{P_S}^0 z\,dp = -\int_0^0 d\,(p\,z) + \int_0^\infty p\,dz = \int_0^\infty \rho\,R\,T\,dz = \frac{R}{C_V}\int_0^\infty \rho\,C_V T\,dz = \frac{R}{C_V}I$$

$$\Rightarrow P \propto I$$

Concept of Available potential energy (APE):

We have seen that in a stably stratified and hydrostatically stable atmosphere I.E. is proportional to the P.E. In such an atmosphere the centre of gravity of the atmosphere is at its lowest elevation. Hence in such condition the atmosphere possesses minimum P.E. and hence it possesses minimum I.E. also. So the sum of these two forms of energy will be minimum in a stably stratified and hydrostatically stable atmosphere. Sum of I.E and P.E. is known as total potential energy (TPE).In the reference state, the atmosphere possesses minimum TPE. In such condition the potential temperature(θ) lines are quasi-horizontal, with θ increasing upwards.

But in a part of the globe, the observed state of atmosphere is not necessarily stably stratified and hydrostatically stable. So, the TPE in the observed configuration exceeds that in reference configuration. The excess TPE in the observed configuration makes the atmosphere unstable. In the observed configuration, the θ lines are quasi-vertical instead of quasi-horizontal, keeping warm air, cold air side by side. As a natural tendency the atmosphere in the observed configuration of that part of the globe tends to be stabilized. This requires rising motion of warm air and sinking motion of cold air, i.e., a vertical circulation is required. The necessary kinetic energy to drive this vertical circulation is provided by the excess TPE in the observed configuration over that in reference configuration. This excess TPE in the observed configuration over that in reference configuration is only available for conversion into kinetic energy and is known as available potential energy (APE).

Thus, $APE = TPE_{OBS} - TPE_{REF}$. It can be shown that, $APE \propto \iint \int_{P_s}^{0} \frac{T'^2}{\overline{P}\sigma} dp\, dx\, dy$, where, σ is a measure of static stability, $\overline{P}$ is the mean pressure at any level and T'^2 is the square of the deviation from mean (areal) temperature at different levels. So, it follows that APE in a barotropic atmosphere is zero. From the above expression of APE, it follows that as the APE over a region increases with the increase in horizontal temperature gradient. So, it's a measure of baroclinity of the atmosphere over that region.